Grade 6 Factors and Multiples

6.N.3	
Demonstrate an understanding of factors and multiples by • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems involving multiples.	1. Identify multiples for a number and explain the strategy used to identify them. [DEVELOPED] 2. Determine all the whole-number factors of a number using arrays. [DEVELOPED] 3. Identify the factors for a number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers or factor trees). [DEVELOPED] 4. Provide an example of a prime number and explain why it is a prime number. [NOT DEVELOPED] 5. Provide an example of a composite number and explain why it is a composite number. [NOT DEVELOPED] 6. Sort a set of numbers as prime and composite. [NOT DEVELOPED] 7. Solve a problem involving factors or multiples. [DEVELOPED] 8. Explain why 0 and 1 are neither prime nor composite. [NOT DEVELOPED]

Clarification of the outcome:

- ✦ The outcome concerns factors (numbers that multiply to give a product) and multiples (numbers on a skip counting list; the result of multiplying two factors). Factors were informally involved in the lesson on primes & composites (the sides of the rectangle are factors of the area number). However, there was no formal development of the concept.
- ✦ Factors can also be thought of as divisors of a number. This viewpoint is used as the starting strategy in the lesson, but it is abandoned later in favor of "finding numbers that multiply to give . . ."

Required close-to-at-hand prior knowledge:

- ✤ Automaticity of multiplication and/or division facts.
- ✤ Understanding primes and composites.

SET SCENE stage

The problem task to present to students:

Have students play the game, ***4-in-a-row***, for 10 to 15 minutes.

Game materials:

- 10 each of two different colored disks
- Pair of dice
- Game board (see diagram)

5	3	7	PRIME
2	6	8	17
PRIME	21	4	16
31	9	PRIME	26

Game rules:

Two players play. The starting player rolls two dice. The largest number rolled is always used for the tens digit and the smallest is always used for the ones digit. For example, if a player rolls a 2 and a 3, the number would be 32 not 23.

The player places **only one disk** during each turn. He/she places it on the playing board over any number that is a divisor of the number he created in his roll. For example, if the number is 32, the player could place one marker on <u>one</u> of these numbers: 2, 4, 8, or 16.

- If a player rolls doubles, the turn is lost.
- If a player rolls a PRIME (e.g. 31), he/she places a marker on any PRIME square.
- If a square is already covered, a player cannot use it.
- The first player to get four disks in a row (horizontal/vertical/diagonal) wins.
- If the grid gets locked and no one can win, players start a new game.

Comments:

The game is designed to lead students to think informally about factors. Using a game for SET SCENE is likely to be a successful strategy for engaging students.

DEVELOP stage

Activity 1: Revisits SET SCENE

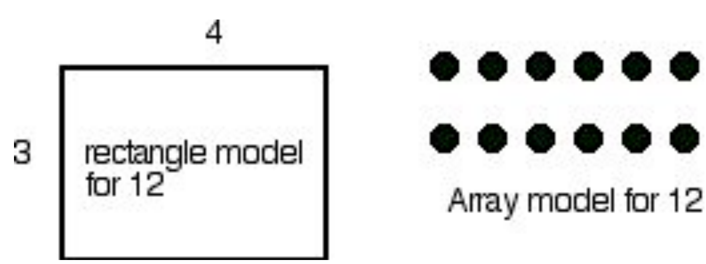
- Discuss the ***4-in-a-row*** game in relation to finding divisors or multipliers of the number rolled. Ask the questions: (1) How did you figure out a divisor/multiplier? and (2) Was there always more than one divisor/multiplier? Discuss student responses. If no student has mentioned the strategy of using primes in turn from smallest to largest possible, introduce students to it. [E.G.: For 24, think 2 x ? = 24 or 24 ÷ 2 = ?, then think 3 x ? = 12 or 12 ÷ 3 = ?, and so on.]
- Present students with the numbers 30, 45, and 58. Ask them to use the strategy of using primes in turn from smallest to largest possible to find the factors (allow calculators). Discuss the results.

Activity 2: Addresses achievement indicator 3.

- ✦ Ask students what are the smallest and largest numbers that can occur in the game, ***4-in-a-row.*** Ensure the they realize the smallest is 21 and the largest is 66.
- ✦ Ask students to select a composite number between 21 and 66. Suppose the students choose '30'. Ask students to write the number using two multipliers where one of the multipliers is prime (e.g. for 30, possibilities are 2 x 15 or 5 x 6 or 3 x 10). Introduce the term 'factor' for the multipliers of 30. Have students find other factors of 30 (include 1 and 30). Discuss the strategy they used for finding a factor.

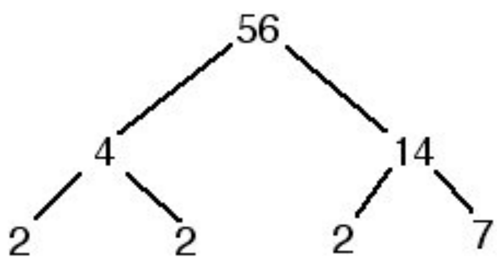
Activity 3: Addresses achievement indicators 2 and 3.

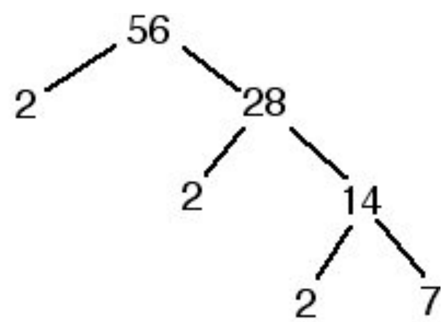
- Present students with the number 12. Ask them to make all possible arrays/rectangles that have an area of 12/dot count of 12. [Refer to the examples.] Ensure all rectangles/arrays are found (1 x 12, 2 x 6, 3 x 4). Discuss the sides of the arrays/rectangles as the factors of 12. Repeat for the numbers 20 and 24.
- Ask students to define the term 'factor'. Discuss their definitions. Steer them in the direction of a factor as a multiplier rather than as a divisor.



Activity 4: Addresses achievement indicators 3 and 7.

- Ask students to select a composite number between 50 and 100. Suppose the students choose '56'. You make a prime factor tree of the number (see examples below for two possibilities). Ask students to describe what you did. Discuss their responses. Ensure that students begin to realize what a prime factor tree is. [You keep finding a prime factor for a branch until you cannot do it anymore, then that branch ends. The resulting primes (2, 2, 2, 7) are the factors that multiply to give the number (56, in this example).]



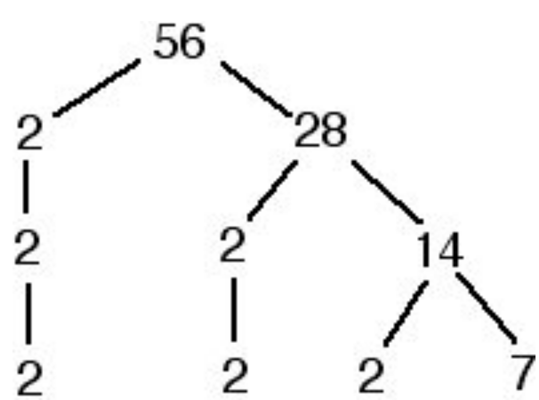


- Present students with the numbers 40, 72, and 85. Ask them to make two different prime factor tress for each number. Discuss the results.

Note:

There are different styles for making a factor tree. The trees above for 56 above are one style where each prime factor is written only once. When a prime is reached, the branch ends at that prime.

Another style is to repeat write a prime factor until all branches end at the bottom (see example). This style is laborious because students have to keep writing the factor(s) to match what happens elsewhere. It also results in a visually cluttered tree that can confuse students. We recommend the shorter style for making a factor tree.



Activity 5: Addresses achievement indicators 3 and 7 (and practice).

- ✦ Provide the list of numbers: 3, 17, 24, 35, 42, 52. Ask students to make a prime factor tree for each number on the list.
- ✦ Ask students to discuss the difference between the trees for the prime numbers on the list (3, 17) and the composite numbers on the list (the rest). Ensure they realize that the prime factor tree for a prime has no branches because a prime has already been reached. Composite numbers have two or more branches.

Note:

This is another visual way to help students understand the distinction between primes and composites (no branches versus at least two branches).

Activity 6: Addresses achievement indicators 1 and 3.

- Show students a skip counting list for 2 (2, 4, 6, 8, . . . up to 20). Ask them to extend the list for three more numbers. Ask them if 2 is a factor of each number on the list. Discuss why. Introduce the term 'multiple' for the numbers on the skip counting list (e.g. 4 is a multiple of 2, 10 is a multiple of 2).
- Show students a skip counting list for 3 (3, 6, 9, . . . up to 24). Ask them to extend the list for three more numbers. Ask them if 3 is a factor of each number on the list. Discuss why. Ask if each number on the list is a multiple of 3. Ask students to define the term 'multiple'. Discuss their definitions. Ensure they realize that a multiple is the result of multiplying factors and that the starting number on the skip counting list is a factor for each multiple (for example: for the first five multiples of 2: ‘2, 4, 6, 8, 10’, ‘2’ is a factor for each number).
- Ask students to list five multiples of 5, 6, and 10. Discuss the results.

Activity 7: Addresses achievement indicators 1 and 3 (and practice).

- ✦ Present the number ’24’ Ask students to list all possible skip counting lists that have 24 in them. [EXPECT: 2, 4, 6, 8, 10, . . . 22, 24; 3, 6, 9, 12, . . . 21, 24; 4, 8, 12, . . . 20, 24; 6, 12, 18, 24; 8, 16, 24; 12, 24). Ask students what numbers ’24’ is a multiple of. [EXPECT: of 2, 3, 4, 6, 8, and 12.]
- ✦ Ask students to list all the numbers that 50 is a multiple of (EXPECT: 2, 5, 10, 25). Discuss the relationship between those numbers and the factors of 50. Ensure students realize that the ‘multiples of’ and the factors are the same, when 1 and 50 are not included as factors.
- ✦ Ask students to list all the numbers that 30 is a multiple of (expect: 2, 3, 5, 6, 10, 15). Discuss the relationship between those numbers and the factors of 30. Ensure students realize that the ‘multiples of’ and the factors are the same, when 1 and 30 are not included as factors.

Activity 8: Addresses achievement indicators 1, 2, and, 3.

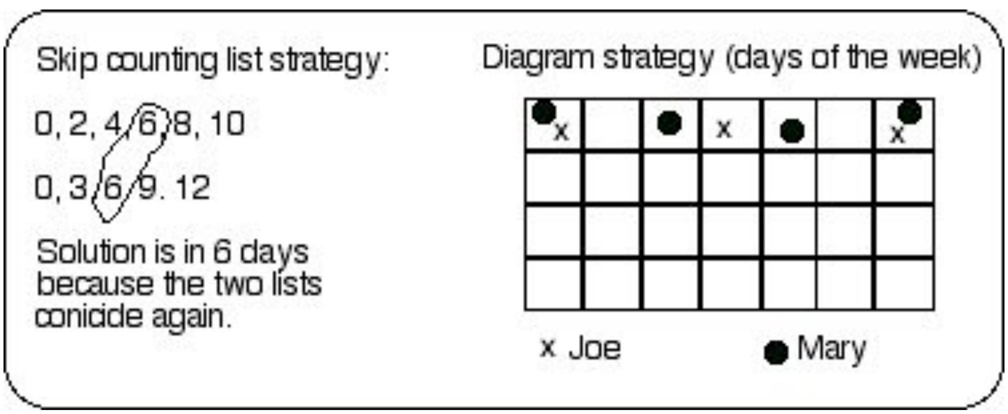
- Revisit one of the array/rectangle models for factors (for example, from activity #3, the model showing 2 x 6 is 12). Ask students which numbers are factors and which number is a multiple. Ensure they realize the sides of the array/rectangle are the factors and the total count of dots/area of rectangle is the multiple. [In the case of 2 x 6 is 12, 2 and 6 are factors of 12 while 12 is a multiple of 2 and a multiple of 6.]
- Repeat for a different array/rectangle model of a number.

Activity 9: Addresses achievement indicators 1, 2, 3, and, 7.

- Present students with problems such as the example below.

  Mary works every second day at 7-11. Joe works at 7-11 every third day. Today they are both working there. In how many days will they work together again at 7-11?

- Have students solve the problems any way they want. Drawing diagrams or making skip counting lists can be helpful (see sample solutions below). Discuss their solution strategies in terms of multiples and factors (e.g. the solution '6' is the first common multiple of the factors 2 and 3).



Activity 10: Addresses achievement indicators 3 and 7, and revisits SET SCENE.

- ✦ Ask students to design a 5 x 5 game board for the game, ***4-in-a-row***. Have students play the game using their game board.

Activity 11: Assessment of teaching.

- Ask students to list three factors of 40. Ask them to explain what a factor is.
- Ask students to list five multiples of 4. Ask them to explain what a multiple is.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

List all the factors of 48.

Select one of the factors and explain why it is a factor.

Question 2.

List the first five multiples of 6.

Select one of the multiples and explain why it is a multiple.

Question 3.

Write the first three numbers less than 30 that have 2 and 5 as factors.

Question 4.

Write a number between 0 and 25 that has:

- six factors
- three factors

Question 5.

Cross out any number on the list that is NOT a multiple of 4:

12, 19, 34, 20, 50, 40, 36

Question 6.

Make a prime factor tree for 28.

Question 7.

A prime factor tree for a number can look differently, depending on how the branches are laid out and on where the factors are placed. Make all possible prime factor trees for 40. [Hint: There are a lot of them. Think about mirror reflections as you make the trees.]

MAINTAIN stage

Mini-task example

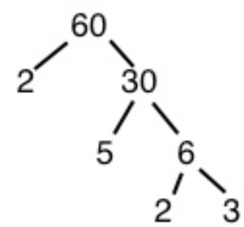
Every so often:

- Provide students with a composite number (between 50 and 100). Ask them to:
  - ★ list all the factors of the number.
  - ★ make a prime factor tree for the number.
  - ★ identify two numbers that the number is a multiple of.
  - ★ list three multiples of the number.

Comment

Suppose 60 is the number.

- ★ list all the factors of the number: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
- ★ make a prime factor tree for the number:



- ★ identify two numbers that the number is a multiple of: 2 and 5
- ★ list three multiples of the number: 120, 180, 240,

Rich-task example

Provide students with the list of numbers: 24, 36, 49, 63, 64, 81, 90, 100, 110, 121, 144, 150. Have them investigate which numbers on the list can be the top number of a perfectly symmetrical prime factor tree ('perfectly symmetrical' means that the number of branches on each side are the same AND the prime factors are identical on each side). Have students come to a conclusion about what kind of numbers can be the top number of a perfectly symmetrical prime factor tree. [Hint: This involves finding a pattern.]

Comment

Students should conclude that only numbers that are the result of multiplying two identical factors (perfect squares, using more sophisticated terminology) can be the top number of a perfectly symmetrical prime factor trees (for the above list, perfectly symmetrical prime factor trees can be made for: 36, 49, 64, 81, 100, 121, 144). [See example for 36.]